UNIVERSITY OF MUMBAI

Syllabus for: T.Y.B.Sc./T.Y.B.A.

Program: B.Sc./B.A.

Course: Mathematics

Choice based Credit System (CBCS)

with effect from the

academic year 2022-23

Syllabus for Approval

Sr. No.	Hearding	Particulars
1	Title of the Course	T.Y.B.Sc. /B.A. Mathematics, Sem V and VI
2	Eligibility for Admission	As per university regulations
3	Passing Marks	40% (Internal 10/25 Marks and External 30/75)
4	Ordinances / Regulations (if any)	-
5	No. of Years/ Semesters	Three Years/ Six Semesters Programme (Syllabus for sem V & VI)
6	Level	UG
7	Pattern	Semester
8	Status	Revised
9	To be implemented from Academic Year:	2022-23

Date: 9th May 2022 **Signature**:
Name: Prof. Vinayak Kulkarni **Chairman of BoS of Mathematics**

Members of the Board of Studies of Mathematics:
Prof. Anuradha Majumdar (Dean, Science and Technology)
Prof. Shivram Garje (Associate Dean, Science)
Prof. R. M. Pawale, Member
Prof. P. Veeramani, Member
Prof. S. R. Ghorpade, Member
Prof. Ajit Diwan, Member
Dr. S. Aggarwal, Member
Dr. Amul Desai, Member
Dr. S. A. Shende, Member
Dr. Shridhar Pawar.
Dr. Sanjeevani Gharge, Member
Dr. Abhaya Chitre, Member
Dr. Mittu Bhattacharya, Member
Dr. Sushil Kulkarni, Member
Dr. Rajiv Sapre, Member

CONTENTS

1. Preamble

The University of Mumbai has brought into force the revised syllabi as per the Choice Based Credit System (CBCS) for the Third year B. Sc / B. A. Programme in Mathematics from the academic year 2022-2023. Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematics to real world problems has increased by leaps and bounds. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects like Physics, Statistics and Computer Sciences, the board of studies in Mathematics with concern of teachers of Mathematics from different colleges affiliated to University of Mumbai has prepared the syllabus of T.Y.B. Sc. / T. Y. B. A. Mathematics. The present syllabi of T. Y. B. Sc. for Semester V and Semester VI has been designed as per U. G. C. Model curriculum so that the students learn Mathematics needed for these branches, learn basic concepts of Mathematics and are exposed to rigorous methods gently and slowly. The syllabi of T. Y. B. Sc. / T. Y. B. A. would consist of two semesters and each semester would comprise of four courses and two practical courses for T. Y. B. Sc / T.Y.B.A. Mathematics.

2. Aims and Objectives:

(i) Give the students a sufficient knowledge of fundamental principles, methods and a clear perception of innumerous power of mathematical ideas and tools and know how to use them by modeling, solving and interpreting.

(ii) Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.

(iii) Enhancing students' overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment.

(iv) A student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences.

3. Programme Outcomes:

(i) Enabling students to develop positive attitude towards mathematics as an interesting and valuable subject

(ii) Enhancing students overall development and to equip them with mathematical modeling, abilities, problem solving skills, creative talent and power of communication.

(iii) Acquire good knowledge and understanding in advanced areas of mathematics and physics.

4. Course outcomes:

(i) **Multivariable Calculus II (Sem V):** In this course students will learn the basic ideas, tools and techniques of integral calculus and use them to solve problems from real-life applications including science and engineering problems involving areas, volumes, centroid, Moments of mass and center of mass Moments of inertia. Examine vector fields and define and evaluate line integrals using the Fundamental Theorem of Line Integrals and Green's Theorem; compute arc length.

(ii) **Complex Analysis (Sem VI):** Students Analyze sequences and series of analytic functions and types of convergence, Students will also be able to evaluate complex contour integrals directly and by the fundamental theorem, apply the Cauchy integral theorem in its various versions, and the Cauchy integral formula, they will also be able to represent functions as Taylor, power and Laurent series, classify singularities and poles, find residues and evaluate complex integrals using the residue theorem.

(iii) **Group Theory, Ring Theory (Sem V, Sem VI)** Students will have a working knowledge of important mathematical concepts in abstract algebra such as definition of a group, order of a finite group and order of an element, rings, Euclidean domain, Principal ideal domain and Unique factorization domain. Students will also understand the connection and transition between previously studied mathematics and more advanced mathematics. The students will actively participate in the transition of important concepts such homomorphisms & isomorphisms from discrete mathematics to advanced abstract mathematics.

(iv) **Topology of metric spaces (Sem V), Topology of metric spaces and real analysis (Sem VI):**
This course introduces students to the idea of metric spaces. It extends the ideas of open sets, closed sets and continuity to the more general setting of metric spaces along with concepts such as compactness and connectedness. Convergence concepts of sequences and series of functions, power series are also dealt with. Formal proofs are given a lot of emphasis in this course. This course serves as a foundation to advanced courses in analysis. Apart from understanding the concepts introduced, the treatment of this course will enable the learner to explain their reasoning about analysis with clarity and rigour.

(v) **Partial Differential equations (Sem V: Paper IV: Elective A):**

a. Students will able to understand the various analytical methods for solving first order partial differential equations.
b. Students will able to understand the classification of first order partial differential equations.
c. Students will able to grasp the linear and non linear partial differential equations.

(vi) **Integral Transforms (Sem VI: Paper IV- Elective A):**

a. Students will able to understand the concept of integral transforms and their corresponding inversion techniques.
b. Students will able to understand the various applications of integral transforms.

(vii) **Number Theory and its applications I and II (Sem V, Sem VI):**
The student will be able to

a. Identify and apply various properties of and relating to the integers including primes, unique factorization, the division algorithm, and greatest common divisors.
b. Understand the concept of a congruence and use various results related to congruences including the Chinese Remainder Theorem. Investigate Pseudo-primes , Carmichael number, primitive roots.
c. Identify how number theory is related to and used in cryptography. Learn to encrypt and decrypt a message using character ciphers. Learn to encrypt and decrypt a message using Public-Key cryptology.
d. Express a rational number as a finite continued fraction and hence solve a linear diophantine equation. Express a given repeated continued fraction in terms of a surd. Expand a surd as an infinite continued fraction and hence find a convergent which is an approximation to the given surd to a given degree of accuracy .Solve a Pell equation from a continued fraction expansion
e. Solve certain types of Diophantine equations. Represent a Primitive Pythagorean Triples with a unique pair of relatively prime integers.
f. Identify certain number theoretic functions and their properties. Investigate perfect numbers and Mersenne prime numbers and their connection. Explore the use of arithmetical functions, the Mobius function, and the Euler function.

(viii) **Graph Theory (Sem V: Paper IV- Elective C)**
Upon successful completion of Graph Theory course, a student will be able to:

a. Demonstrate the knowledge of fundamental concepts in graph theory, including properties and characterization of graphs and trees.
b. Describe knowledgeably special classes of graphs that arise frequently in graph theory
c. Describe the concept of isomorphic graphs and isomorphism invariant properties of graphs
d. Describe and apply the relationship between the properties of a matrix representation of a graph and the structure of the underlying graph
e. Demonstrate different types of algorithms including Dijkstra's, BFS, DFS, MST and Huffman coding.
f. Understand the concept of Eulerian graphs and Hamiltonian graphs.
g. Describe real-world applications of graph theory.

(ix) **Graph Theory and Combinatorics (Sem VI: Paper IV -Elective C)**

a. Understand and apply the basic concepts of graph theory, including colouring of graph, to find chromatic number and chromatic polynomials for graphs
b. Understand the concept of vertex connectivity, edge connectivity in graphs and Whitney's theorem on 2-vertex connected graphs.
c. Derive some properties of planarity and Euler's formula, develop the under-standing of Geometric duals in Planar Graphs
d. Know the applications of graph theory to network flows theory.
e. Understand different applications of system of distinct representative and matching theory.
f. Use permutations and combinations to solve counting problems with sets and multisets.
g. Set up and solve a linear recurrence relation and apply the inclusion/exclusion principle.
h. Compute a generating function and apply them to combinatorial problems.

(x) **Basic concepts of probability and random variables (Sem V: Paper IV: Elective D)**
Students will be able to understand the role of random variables in the statistical analysis and use them to apply in the various probability distributions including Binomial distribution, Poisson distribution and Normal distribution. Moreover students will able to apply the concepts of expectations and moments for the evaluation of various statistical measures

(xi) **Operations research (Sem VI: Paper IV: Elective D)**
Students should able to formulate linear programming problem and apply the graphical and simplex method for their feasible solution. Moreover students should understand various alternative operation research techniques for the feasible solution of LPP.

(5) Course structure with minimum credits and Lectures/ Week

SEMESTER V

Course Code	UNIT	TOPICS	Credits	L/Week
Multivariable Calculus II				
USMT 501, UAMT 501	I	Multiple Integrals	2.5	3
	II	Line Integrals		
	III	Surface Integrals		
Group Theory				
USMT 502 ,UAMT 502	I	Groups and Subgroups	2.5	3
	II	Normal subgroups, Direct products and Cayley's theorem		
	III	Cyclic Groups and Cyclic Subgroups Homomorphism		
Topology of Metric Spaces				
USMT 503, UAMT503	I	Metric spaces	2.5	3
	II	Sequences and Complete metric spaces		
	III	Compact Spaces		
Partial Differential Equations(Elective A)				
USMT5A4 ,UAMT 5A4	I	First Order Partial Differential Equations.	2.5	3
	II	Compatible system of first order PDE		
	III	Quasi-Linear PDE		
Number Theory and Its applications I (Elective B)				
USMT5B4 ,UAMT 5B4	I	Congruences and Factorization	2.5	3
	II	Diophantine equations and their & solutions		
	III	Primitive Roots and Cryptography		
Graph Theory (Elective C)				
USMT5C4 ,UAMT 5C4	I	Basics of Graphs	2.5	3
	II	Trees		
	III	Eulerian and Hamiltonian graphs		
Basic Concepts of Probability and Random Variables (Elective D)				
USMT5D4 ,UAMT 5D4	I	Basic Concepts of Probability and Random Variables	2.5	3
	II	Properties of Distribution function, Joint Density function		
	III	Weak Law of Large Numbers		
PRACTICALS				
USMTP05/UAMTP05		Practicals based on USMT501/UAMT 501 and USMT 502/UAMT 502	3	6
USMTP06/UAMTP06		Practicals based on USMT503/ UAMT 503 and USMT5A4/ UAMT 5A4 OR USMT5B4/ UAMT 5B4 OR USMT5C4/ UAMT 5C4 OR USMT5D4/ UAMT 5D4	3	6

SEMESTER VI

BASIC COMPLEX ANALYSIS				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 601, UAMT 601	I	Introduction to Complex Analysis	2.5	3
	II	Cauchy Integral Formula		
	III	Complex power series, Laurent series and isolated singularities		
Ring Theory				
USMT 602 ,UAMT 602	I	Rings	2.5	3
	II	Ideals and special rings		
	III	Factorization		
Topology of Metric Spaces and Real Analysis				
USMT 603 / UAMT 603	I	Continuous functions on Metric spaces	2.5	3
	II	Connected sets		
		Sequences and series of functions		
Integral Transforms(Elective A)				
USMT6A4 ,UAMT 6A4	I	The Laplace Transform	2.5	3
	II	The Fourier Transform		
	III	Applications of Integral Transforms		
Number Theory and Its applications II (Elective B)				
USMT6B4 ,UAMT 6B4	I	Quadratic Reciprocity	2.5	3
	II	Continued Fractions		
	III	Pell's equation, Arithmetic function & and Special numbers		
Graph Theory and Combinatorics (Elective C)				
USMT6C4 ,UAMT 6C4	I	Colourings of Graphs	2.5	3
	II	Planar graph		
	III	Combinatorics		
Operations Research (Elective D)				
USMT6D4 ,UAMT 6D4	I	Basic Concepts of Probability and Linear Programming I	2.5	3
	II	Linear Programming II		
	III	Queuing Systems		
PRACTICALS				
USMTP07/ UAMTP07		Practicals based on USMT601/UAMT 601 and USMT 602/UAMT 602	3	6
USMTP08/UAMTP08		Practicals based on USMT603/ UAMT 603 and USMT6A4/ UAMT 6A4 OR USMT6B4/ UAMT 6B4 OR USMT6C4/ UAMT 6C4 OR USMT6D4/ UAMT 6D4	3	6

Note:

i . USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503 are compulsory courses for Semester V.

ii . Candidate has to opt one Elective Course from USMT5A4/UAMT5A4, USMT5B4/UAMT5B4, USMT5C4/UAMT5C4 and USMT5D4/UAMT5D4 for Semester V.

iii . USMT601/UAMT601, USMT602/UAMT602, USMT603/UAMT603 are compulsory courses for Semester VI.

iv . Candidate has to opt one Elective Course from USMT6A4/UAMT6A4, USMT6B4/UAMT6B4, USMT6C4/UAMT6C4 and USMT6D4/UAMT6D4 for Semester VI.

v . Passing in theory and practical and internal exam shall be separate.

(6) Teaching Pattern for T.Y.B.Sc/B.A.

i. Three lectures per week per course (1 lecture/period is of 48 minutes duration).

ii. One practical of three periods per week per course (1 lecture/period is of 48 minutes duration).

(7)Consolidated Syllabus for semester V & VI

SEMESTER V
MULTIVARIABLE CALCULUS II
Course Code: USMT501/UAMT501

ALL Results have to be done with proof unless otherwise stated.

Unit I: Multiple Integrals (15L)

Definition of double (resp: triple) integral of a function and bounded on a rectangle (resp:box). Geometric interpretation as area and volume. Fubini's Theorem over rectangles and any closed bounded sets, Iterated Integrals. Following basic properties of double and triple integrals proved using the Fubini's theorem:

(1) Integrability of the sums, scalar multiples, products, and (under suitable conditions) quotients of integrable functions. Formulae for the integrals of sums and scalar multiples of integrable functions.

(2) Integrability of continuous functions. More generally, Integrability of functions with a "small" set of (Here, the notion of "small sets" should include finite unions of graphs of continuous functions.)

(3) Domain additivity of the integral. Integrability and the integral over arbitrary bounded domains. Change of variables formula (Statement only).Polar, cylindrical and spherical coordinates, and integration using these coordinates. Differentiation under the integral sign. Applications to finding the center of gravity and moments of inertia.

Unit 2: Line Integrals (15L)

Review of Scalar and Vector fields on $\mathbb{R}^n$, Vector Differential Operators, Gradient, Curl, Divergence.
Paths (parametrized curves) in $\mathbb{R}^n$ (emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$), Smooth and piecewise smooth paths. Closed paths. Equivalence and orientation preserving equivalence of paths. Definition of the line integral of a vector field over a piecewise smooth path. Basic properties of line integrals including linearity, path-additivity and behaviour under a change of parameters. Examples.
Line integrals of the gradient vector field, Fundamental Theorem of Calculus for Line Integrals, Necessary and sufficient conditions for a vector field to be conservative. Green's Theorem (proof in the case of rectangular domains). Applications to evaluation of line integrals.

Unit 3: Surface Integrals (15 L)

Parameterized surfaces. Smoothly equivalent parameterizations. Area of such surfaces.
Definition of surface integrals of scalar-valued functions as well as of vector fields defined on a surface. Curl and divergence of a vector field. Elementary identities involving gradient, curl and divergence. Stoke's Theorem (proof assuming the general from of Green's Theorem). Examples.
Gauss' Divergence Theorem (proof only in the case of cubical domains). Examples.
Reference Books:

1. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969 Section 1.1 to 11.8

2. James Stewart, Calculus with early transcendental Functions - Section 16.5 to 16.9

3. Marsden and Jerrold E. Tromba, Vector Calculus, Fourth Ed., W.H. Freeman and Co., New York, 1996 Section 6.2 to 6.4.

Other References :

1. T. Apostol, Mathematical Analysis, Second Ed., Narosa, New Delhi. 1947.
2. R. Courant and F.John, Introduction to Calculus and Analysis, Vol.2, Springer Verlag, New York, 1989.
3. W. Fleming, Functions of Several Variables, Second Ed., Springer-Verlag, New York, 1977.
4. M. H. Protter and C.B.Morrey Jr., Intermediate Calculus, Second Ed., Springer-Verlag, New York, 1995.
5. G. B. Thomas and R.L Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1998.
6. D. V. Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989.

Course: Group Theory
Course Code: USMT502/UAMT502

Unit 1: Groups and Subgroups (15L)

(1) Definition and elementary properties of a group. Order of a group. Subgroups. Criterion for a subset to be a subgroup. Abelian groups. Center of a group. Homomorphisms and isomorphisms.

(2) Examples of groups including $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$, Klein 4-group, symmetric and alternating groups, S^1(= the unit circle in $\mathbb{C}$), $\mathrm{GL}_n(\mathbb{R}), \mathrm{SL}_n(\mathbb{R}), \mathrm{O}_n$ (= the group of $n \times n$ nonsingular upper triangular matrices), B_n (= the group of $n \times n$ nonsingular upper triangular matrices), and groups of symmetries of plane figures.

(3) Order of an element. Subgroup generated by a subset of the group.

Unit 2: Normal subgroups, Direct products and Cayley's Theorem (15L)

(1) Cosets of a subgroup in a group. Lagrange's Theorem. Normal subgroups. Alternating group A_n. Listing normal subgroups of A_4, S_3. Quotient (or Factor) groups. Fundamental Theorem of homomorphisms of groups.

(2) External direct products of groups. Examples. Relation with internal products such as HK of subgroups H, K of a group.

(3) Cayley's Theorem for finite groups.

Unit 3: Cyclic groups and cyclic subgroups (15L)

(1) Examples of cyclic groups such as $\mathbb{Z}$ and the group μ_n of the $n-$th roots of unity. Properties of cyclic groups and cyclic subgroups.

(2) Finite cyclic groups, infinite cyclic groups and their generators. Properties of generators.

(3) The group $\mathbb{Z}/n\mathbb{Z}$ of residue classes (mod n). Characterization of cyclic groups (as being isomorphic to $\mathbb{Z}$ or $\mathbb{Z}/n\mathbb{Z}$ for some $n \in \mathbb{N}$).

Recommended Books.

1. I. N. Herstein, Topics in Algebra, Wiley Eastern Limied, Second edition.
2. P. B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
3. N. S. Gopalkrishnan, University Algebra, Wiley Eastern Limited.
4. M. Artin, Algebra, Prentice Hall of India, New Delhi.
5. J. B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.

Additional Reference Books

1. T. W. Hungerford. Algebra, Springer.
2. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
3. I. S. Luther, I.B.S. Passi. Algebra. Vol. I and II.

Course: Topology of Metric Spaces
Course Code: USMT503/UAMT503

Unit I: Metric spaces (15 L)
Definition and examples of metric spaces such as $\mathbb{R}, \mathbb{R}^2$, $\mathbb{R}^n$ with its Euclidean, sup and sum metrics. $\mathbb{C}$ (complex numbers). l^1 and l^2 spaces of sequences. $C[a, b]$ the space of real valued continuous functions on $[a, b]$. Discrete metric space. Metric induced by the norm. Translation invariance of the metric induced by the norm. Metric subspaces. Product of two metric spaces. Open balls and open sets in a metric space. Examples of open sets in various metric spaces. Hausdorff property. Interior of a set. Properties of open sets. Structure of an open set in $\mathbb{R}$. Equivalent metrics.
Distance of a point from a set, Distance between sets. Diameter of a set. Bounded sets. Closed balls. Closed sets. Examples. Limit point of a set. Isolated point. Closure of a set. Boundary of a set.

Unit II: Sequences and Complete metric spaces (15L)
Sequences in a metric space. Convergent sequence in metric space. Cauchy sequence in a metric space. Subsequences. Examples of convergent and Cauchy sequences in different metric spaces. Characterization of limit points and closure points in terms of sequences. Definition and examples of relative openness/closeness in subspaces. Dense subsets in a metric space and Separability. Definition of complete metric spaces. Examples of complete metric spaces. Completeness property in subspaces. Nested Interval theorem in $\mathbb{R}$. Cantor's Intersection Theorem. Applications of Cantors Intersection Theorem:

(i) The set of real Numbers is uncountable.

(ii) Density of rational Numbers.

(iii) Intermediate Value theorem.

Unit III: Compact spaces (15L)
Definition of a compact metric space using open cover. Examples of compact sets in different metric spaces such as $\mathbb{R}, \mathbb{R}^2, \mathbb{R}^n$ with Euclidean metric. Properties of compact sets: A compact set is closed and bounded, (Converse is not true). Every infinite bounded subset of compact metric space has a limit point. A closed subset of a compact set is compact. Union and Intersection of Compact sets.
Equivalent statements for compact sets in $\mathbb{R}$ with usual metric:

(i) Sequentially compactness property.

(ii) Heine-Borel property.

(iii) Closed and boundedness property.

(iv) Bolzano-Weierstrass property.

Reference books:

1. S. Kumaresan; Topology of Metric spaces.
2. E. T. Copson; Metric Spaces; Universal Book Stall, New Delhi, 1996.
3. P. K. Jain, K. Ahmed; Metric Spaces; Narosa, New Delhi, 1996.

Other references :

1. T. Apostol; Mathematical Analysis, Second edition, Narosa, New Delhi, 1974
2. R. R. Goldberg; Methods of Real Analysis; Oxford and IBH Pub. Co., New Delhi 1970.
3. D. Gopal, A. Deshmukh, A. S. Ranadive and S. Yadav; An Introduction to Metric Spaces, Chapman and Hall/CRC, New York, 2020.
4. W. Rudin; Principles of Mathematical Analysis; Third Ed, McGraw-Hill, Auckland, 1976.
5. D. Somasundaram; B. Choudhary; A first Course in Mathematical Analysis. Narosa, New Delhi
6. G. F. Simmons; Introduction to Topology and Modern Analysis; McGraw-Hi, New York, 1963.
7. Expository articles of MTTS programme.

Course: Partial Differential Equations (Elective A)
Course Code: USMT5A4/UAMT5A4

Unit I: First Order Partial Differential Equations. (15L)

Curves and Surfaces, Genesis of first order PDE, Classification of first order PDE, Classification of integrals, The Cauchy problem, Linear Equation of first order, Lagrange's equation, Pfaffian differential equations. (Ref Book: An Elementary Course in Partial Differential Equations by T. Amaranath, 2nd edition, Chapter 1: 1.1, 1.2, 1.3, Lemma 1.3.1, 1.3.2, 1.3.3, 1.4, Theorem

1.4.1, 1.4.2, 1.5, Theorem 1.5.1, Lemma 1.5.1, Theorem 1.5.2, Lemma 1.5.2 and related examples)

Unit II: Compatible system of first order Partial Differential Equations. (15L)

Definition, Necessary and sufficient condition for integrability, Charpit's method, Some standard types, Jacobi's method, The Cauchy problem. (Ref Book: An Elementary Course in Partial Differential Equations by T. Amaranath, 2nd edition, Chapter 1: 1.6, Theorem 1.6.1, 1.7, 1.8 Theorem 1.8.1, 1.9 and related examples)

Unit III: Quasi-Linear Partial Differential Equations. (15L)

Semi linear equations, Quasi-linear equations, first order quasi-linear PDE, Initial value problem for quasi-linear equation, Non linear first order PDE, Monge cone, Analytic expression for Monge's cone, Characteristics strip, Initial strip. (Ref Book: An Elementary Course in Partial Differential Equations by T. Amaranath, 2nd edition, Chapter 1: 1.10, Theorem 1.10.1, 1.11, Theorem 1.11.1, Preposition 1.11.1, 1.11.2 and related examples)

Reference Books

1. T. Amaranath; An Elementary Course in Partial Differential Equations; 2nd edition, Narosa Publishing house.

2. Ian Sneddon; Elements of Partial Differential Equations; McGraw Hill book.

3. Ravi P. Agarwal and Donal O'Regan; Ordinary and Partial Differential Equations; Springer, First Edition (2009).

4. W. E. Williams; Partial Differential Equations; Clarendon Press, Oxford, (1980).

5. K. Sankara Rao; Introduction to Partial Differential Equations; Third Edition, PHI.

Course: Number Theory and its applications I (Elective B)
Course Code: USMT5B4 / UAMT5B4

Unit I: Congruences and Factorization (15L)

Review of Divisibility, Primes and The fundamental theorem of Arithmetic.
Congruences : Definition and elementary properties, Complete residue system modulo m, Reduced residue system modulo m , Euler's function and its properties, Fermat's little Theorem, Euler's generalization of Fermat's little Theorem, Wilson's theorem, Linear congruence, The Chinese remainder Theorem, Congruences of Higher degree,

Unit II: Diophantine equations and their solutions (15L)

The linear equations $ax + by = c$. The equations $x^2 + y^2 = p$, where p is a prime. The equation $x^2 + y^2 = z^2$, Pythagorean triples, primitive solutions, The equations $x^4 + y^4 = z^2$ and $x^4 + y^4 = z^4$ have no solutions $(x; y; z)$ with $xyz \neq 0$. Every positive integer n can be expressed as sum of squares of four integers, Universal quadratic forms $x^2 + y^2 + z^2 + t^2$. Assorted examples

:section 5.4 of Number theory by Niven- Zuckermann-Montgomery.

Unit III: Primitive Roots and Cryptography (15L)

Order of an integer and Primitive Roots. Basic notions such as encryption (enciphering) and decryption (deciphering), Cryptosystems, symmetric key cryptography, Simple examples such as shift cipher, Affine cipher, Hill cipher, Vigenere cipher. Concept of Public Key Cryptosystem; RSA Algorithm. An application of Primitive Roots to Cryptography.

Reference Books:

1. Niven, H. Zuckerman and H. Montogomery; An Introduction to the Theory of Numbers; John Wiley & Sons. Inc.
2. David M. Burton; An Introduction to the Theory of Numbers; Tata McGrawHillll Edition.
3. G. H. Hardy and E.M. Wright; An Introduction to the Theory of Numbers; Low priced edition; The English Language Book Society and Oxford University Press, 1981.
4. Neville Robins. Beginning Number Theory; Narosa Publications.
5. S.D. Adhikari; An introduction to Commutative Algebra and Number Theory; Narosa Publishing House.
6. N. Koblitz; A course in Number theory and Cryptography; Springer.
7. M. Artin; Algebra; Prentice Hall.
8. K. Ireland, M. Rosen; A classical introduction to Modern Number Theory; Second edition, Springer Verlag.
9. William Stalling; Cryptology and network security.

Course: Graph Theory (Elective C)
Course Code: USMT5C4/UAMT5C4

Unit I: Basics of Graphs (15L)

Definition of general graph, Directed and undirected graph, Simple and multiple graph, Types of graphs- Complete graph, Null graph, Complementary graphs, Regular graphs Sub graph of a graph, Vertex and Edge induced sub graphs, Spanning sub graphs. Basic terminology- degree of a vertex, Minimum and maximum degree, Walk, Trail, Circuit, Path, Cycle. Handshaking theorem and its applications, Isomorphism between the graphs and consequences of isomorphism between the graphs, Self complementary graphs, Connected graphs, Connected components. Matrices associated with the graphs – Adjacency and Incidence matrix of a graph- properties, Bipartite graphs and characterization in terms of cycle lengths. Degree sequence and Havel-Hakimi theorem, Distance in a graph- shortest path problems, Dijkstra's algorithm.

Unit II: Trees (15L)

Cut edges and cut vertices and relevant results, Characterization of cut edge, Definition of a tree and its characterizations, Spanning tree, Recurrence relation of spanning trees and Cayley formula for spanning trees of K_n , Algorithms for spanning tree-BFS and DFS, Binary and m-ary tree, Prefix codes and Huffman coding, Weighted graphs and minimal spanning trees - Kruskal's algorithm for minimal spanning trees.

Unit III: Eulerian and Hamiltonian graphs (15L)

Eulerian graph and its characterization- Fleury's Algorithm-(Chinese postman problem), Hamiltonian graph, Necessary condition for Hamiltonian graphs using $G \setminus S$ where S is a proper subset of $V(G)$, Sufficient condition for Hamiltonian graphs- Ore's theorem and Dirac's theorem, Hamiltonian closure of a graph, Cube graphs and properties like regular, bipartite, Connected and Hamiltonian nature of cube graph, Line graph of graph and simple results.

Reference Books:

1. Bondy and Murty; Grapgh Theory with Applications.
2. Balkrishnan and Ranganathan; Graph theory and applications.
3. Douglas B. West, Introduction to Graph Theory, 2nd Ed., Pearson, 2000

Additional Reference Book:

1. Behzad and Chartrand; Graph theory.
2. Choudam S. A.; Introductory Graph theory.

Course: Basic Concepts of Probability and Random Variables (Elective D)
Course Code: USMT5D4 / UAMT5D4

Unit I: Basic Concepts of Probability and Random Variables.(15 L)

Basic Concepts: Algebra of events including countable unions and intersections, Sigma field $\mathcal{F}$, Probability measure$P$ on $\mathcal{F}$, Probability Space as a triple $(\Omega, \mathcal{F}, P)$, Properties of P including Subadditivity. Discrete Probability Space, Independence and Conditional Probability, Theorem of Total Probability. Random Variable on $(\Omega, \mathcal{F}, P)$ – Definition as a measurable function, Classification of random variables - Discrete Random variable, Probability function, Distribution function, Density function and Probability measure on Borel subsets of $\mathbb{R}$, Absolutely continuous random variable. Function of a random variable; Result on a random variable R with distribution function F to be absolutely continuous, Assume F is continuous everywhere and has a continuous derivative at all points except possibly at finite number of points, Result on density function f_2 of R_2 where $R_2 = g(R_1), h_j$ is inverse of g over a 'suitable' subinterval $f_2(y) + \sum_{i=1}^{n} f_1(h_j(y))|h'_j(y)|$ under suitable conditions.
Reference for Unit 1, Sections 1.1-1.6, 2.1-2.5 of Basic Probability theory by Robert Ash, Dover Publication, 2008.

Unit II: Properties of Distribution function, Joint Density function (15L)

Properties of distribution function F, F is non-decreasing, $\lim_{x \longrightarrow \infty} F(x) = 1$, $\lim_{x \longrightarrow -\infty} F(x) = 0$, Right continuity of F, $\lim_{x \longrightarrow x_0} F(x) = P(\{R < x_o\}, P(\{R = x_o\}) = F(x_o) - F(\overline{x_0})$. Joint distribution, Joint Density, Results on Relationship between Joint and Individual densities, Related result for Independent random variables. Examples of distributions like Binomial, Poisson and Normal distribution. Expectation and $k-$th moments of a random variable with properties.

Reference for Unit II:
Sections 2.5-2.7, 2.9, 3.2-3.3,3.6 of Basic Probability theory by Robert Ash, Dover Publication, 2008.

Unit III: Weak Law of Large Numbers

Joint Moments, Joint Central Moments, Schwarz Inequality, Bounds on Correlation Coefficient ρ ,Result on ρ as a measure of linear dependence, $\mathrm{Var}\Big(\sum_{i=1}^{n} R_i\Big) = \sum_{i=1}^{n} Var(R_i)+2 \sum_{i=1 \leq i<j \leq n}^{n} \mathrm{Cov}(R_i, R_j)$, Method of Indicators to find expectation of a random variable, Chebyshev's Inequality, Weak law of Large numbers.

Reference for Unit III
Sections 3.4, 3.5, 3.7, 4.1-4.4 of Basic Probability theory by Robert Ash, Dover Publication, 2008.

Additional Reference Books. Marek Capinski, Probability through Problems, Springer.

Course: Practicals (Based on USMT501 / UAMT501 and USMT502 / UAMT502)
Course Code: USMTP05 / UAMTP05

Suggested Practicals (Based on USMT501 / UAMT501)

1. Evaluation of double and triple integrals.
2. Change of variables in double and triple integrals and applications
3. Line integrals of scalar and vector fields
4. Green's theorem, conservative field and applications
5. Evaluation of surface integrals
6. Stoke's and Gauss divergence theorem
7. Miscellaneous theory questions on units 1, 2 and 3.

Suggested Practicals (Based on USMT502 / UAMT502)

1. Examples of groups and groups of symmetries of equilateral triangle, square and rectangle.
2. Examples of determining centers of different groups. Examples of subgroups of various groups and orders of elements in a group.
3. Left and right cosets of a group and Lagrange's theorem.
4. Normal subgroups and quotient groups. Direct products of groups.
5. Finite cyclic groups and their generators

6. Infinite cyclic groups and their properties.
7. Miscellaneous Theory Questions

Course: Practicals (Based on USMT503 / UAMT503 and USMT5A4 OR USMT5B4 OR USMT5C4 OR USMT5D4)
Course Code: USMTP06 / UAMTP06

Suggested Practicals USMT503 / UAMT503:

1. Examples of Metric Spaces, Normed Linear Spaces,
2. Sketching of Open Balls in $\mathbb{R}^2$, Open and Closed sets, Equivalent Metrics
3. Subspaces, Interior points, Limit Points, Dense Sets and Separability, Diameter of a set, Closure.
4. Limit Points ,Sequences , Bounded , Convergent and Cauchy Sequences in a Metric Space.
5. Complete Metric Spaces and Applications.
6. Examples of Compact Sets.
7. Miscellaneous Theory Questions.

Suggested Practicals on USMT5A4/UAMT5A4

1. Find general solution of Langrange's equation.
2. Show that Pfaffian differential equation are exact and find corresponding integrals.
3. Find complete integral of first order PDE using Charpit's Method.
4. Find complete integral using Jacobi's Method.
5. Solve initial value problem for quasi-linear PDE.
6. Find the integral surface by the method of characteristics.
7. Miscellaneous Theory Questions.

Suggested Practicals based on USMT5B4/UAMT5B4

1. Congruences.
2. Linear congruences and congruences of Higher degree.
3. Linear diophantine equation.
4. Pythagorean triples and sum of squares.
5. Cryptosystems (Private Key).
6. Cryptosystems (Public Key) and primitive roots.
7. Miscellaneous theoretical questions based on full USMT5B4 .

Suggested Practicals based on USMT5C4/UAMT5C4

1. Handshaking Lemma and Isomorphism.
2. Degree sequence and Dijkstra's algorithm
3. Trees, Cayley Formula
4. Applications of Trees
5. Eulerian Graphs.
6. Hamiltonian Graphs.
7. Miscellaneous Problems.

Suggested Practicals based on USMT5D4/UAMT5D4

1. Basic concepts of Probability (Algebra of events, Probability space, Probability measure, combinatorial problems)
2. Conditional Probability, Random variable (Independence of events. Definition, Classification and function of a random variable)
3. Distribution function, Joint Density function.
4. Expectation of a random variable, Normal distribution.
5. Method of Indicators, Weak law of large numbers.
6. Conditional density, Conditional expectation.
7. Miscellaneous Theoretical questions based on full paper.

SEMESTER VI
BASIC COMPLEX ANALYSIS
Course Code: USMT601/UAMT601

Unit I: Introduction to Complex Analysis (15 L)
Review of complex numbers: Complex plane, polar coordinates, exponential map, powers and roots of complex numbers, De Moivre's formula, $\mathbb{C}$ as a metric space, bounded and unbounded sets, point at infinity-extended complex plane, sketching of set in complex plane (No questions to be asked).
convergence of sequences of complex numbers and related results. Limit of a function $f : \mathbb{C} \longrightarrow \mathbb{C}$, real and imaginary part of functions, continuity at a point and algebra of continuous functions. Derivative of $f : \mathbb{C} \longrightarrow \mathbb{C}$, comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient conditions for differentiability, analytic function, if f, g analytic then $f + g, f - g, fg$ and f/g are analytic, chain rule.
Theorem: If $f(z) = 0$ everywhere in a domain D, then $f(z)$ must be constant throughout D.
Harmonic functions and harmonic conjugate.

Unit II: Cauchy Integral Formula (15 L)
Evaluation the line integral $\int f(z)\ dz$ over $|z - z_0| = r$ and Cauchy integral formula.

Taylor's theorem for analytic function. Mobius transformations: definition and examples. Exponential function, its properties. trigonometric functions and hyperbolic functions.

Unit III: Complex power series, Laurent series and isolated singularities. (15 L)
Power series of complex numbers and related results. Radius of convergences, disc of convergence, uniqueness of series representation, examples.
Definition of Laurent series , Definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighbourhood of an isolated singularity, type of isolated singularities viz. removable, pole and essential defined using Laurent series expansion, examples Statement of Residue theorem and calculation of residue.

Reference Books:

1. J.W. Brown and R.V. Churchill, Complex analysis and Applications : Sections 18, 19, 20, 21, 23, 24, 25, 28, 33, 34, 47, 48, 53, 54, 55 , Chapter 5, page 231 section 65, define residue of a function at a pole using Theorem in section 66 page 234, Statement of Cauchy's residue theorem on page 225, section 71 and 72 from chapter 7.

Other References:

1. Robert E. Greene and Steven G. Krantz, Function theory of one complex variable
2. T.W. Gamelin, Complex analysis

Course: Ring Theory
Course Code: USMT602 / UAMT602

Unit I. Rings (15L)

(1) Definition and elementary properties of rings (where the definition should include the existence of unity), commutative rings, integral domains and fields. Examples, including $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{Z}/n\mathbb{Z}, \mathbb{C}, M_n(\mathbb{R}), \mathbb{Z}[i], \mathbb{Z}[\sqrt{2}], \mathbb{Z}[\sqrt{-5}], \mathbb{Z}[X], \mathbb{R}[X], \mathbb{C}[X], (\mathbb{Z}/n\mathbb{Z})[X]$.

(2) Units in a ring. The multiplicative group of units in a ring R [and, in particular, the multiplicative group F^* of nonzero elemets of a field F]. Description of the units in $\mathbb{Z}/n\mathbb{Z}$. Results such as: A finite integral domain is a field. $\mathbb{Z}/p\mathbb{Z}$, where p is a prime, as an example of a finite field.

(3) Characteristic of a ring. Examples. Elementary facts such as: the characteristic of an itegral domain is either 0 or a prime number.

(Note: From here on all rings are assumed to be commutative with unity).

Unit II. Ideals and special rings(15L)

(1) Ideals in a ring. Sums and products of ideals. Quotient rings. Examples. Prime ideals and maximal ideals. Characterization of prime ideals and maximal ideals in a commutative ring in terms of their quotient rings. Description of the ideals and the prime ideals in $\mathbb{Z}, \mathbb{R}[X]$ and $\mathbb{C}[X]$.

(2) Homomorphisms and isomorphism of rings. Kernel and the image of a homomorphism. Fundamental Theorem of homomorphism of a ring.

(3) Construction of the quotient field of an integral domain (Emphasis on $\mathbb{Z}, \mathbb{Q}$). A field contains a subfield isomorphic to $\mathbb{Z}/p\mathbb{Z}$ or $\mathbb{Q}$.

(4) Notions of euclidean domain (ED), principal ideal domain (PID). Examples such as $\mathbb{Z}, \mathbb{Z}[i]$, and polynomial rings. Relation between these two notions (ED $\implies$ PID).

Unit III. Factorization (15L)

(1) Divisibility in a ring. Irreducible and prime elements. Examples.

(2) Division algorithm in $F[X]$ (where F is a field). Monic polynomials, greatest common divisor of $f(x), g(x) \in F[X]$ (not both 0). Theorem: Given $f(x)$ and $g(x) \neq 0$, in $F[X]$ then their greatest common divisor $d(x) \in F[X]$ exists; moreover, $d(x) = a(x)f(x) + b(x)g(x)$ for some $a(x), b(x) \in F[X]$. Relatively prime polynomials in $F[X]$, irreducible polynomial in $F[X]$. Examples of irreducible polynomials in $(\mathbb{Z}/p\mathbb{Z})[X]$ (p prime), Eisenstein Criterion (without proof).

(3) Notion of unique factorization domain (UFD). Elementary properties. Example of a non-UFD is $\mathbb{Z}[\sqrt{-5}]$ (without proof). Theorem (without proof). Relation between the three notions (ED $\implies$ PID $\implies$ UFD). Examples such as $\mathbb{Z}[X]$ of UFD that are not PID. Theorem (without proof): If R is a UFD, then $R[X]$ is a UFD.

Reference Books

1. N. Herstein; Topics in Algebra; Wiley Eastern Limited, Second edition.
2. P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul; Abstract Algebra; Second edition, Foundation Books, New Delhi, 1995.
3. N. S. Gopalakrishnan; University Algebra; Wiley Eastern Limited.
4. M. Artin; Algebra; Prentice Hall of India, New Delhi.
5. J. B. Fraleigh; A First course in Abstract Algebra; Third edition, Narosa, New Delhi.
6. J. Gallian; Contemporary Abstract Algebra; Narosa, New Delhi.

Additional Reference Books:

1. S. Adhikari; An Introduction to Commutative Algebra and Number theory; Narosa Publishing House.
2. T.W. Hungerford; Algebra; Springer.
3. D. Dummit, R. Foote; Abstract Algebra; John Wiley & Sons, Inc.
4. I.S. Luthar, I.B.S. Passi; Algebra; Vol. I and II.
5. U. M. Swamy, A. V. S. N. Murthy; Algebra Abstract and Modern; Pearson.
6. Charles Lanski; Concepts Abstract Algebra; American Mathematical Society.
7. Sen, Ghosh and Mukhopadhyay; Topics in Abstract Algebra; Universities press.

Course: Topology of Metric Spaces and Real Analysis
Course Code: USMT603/ UAMT603

Unit I: Continuous functions on metric spaces (15 L)

Epsilon-delta definition of continuity of a function at a point from one metric space to another. Characterization of continuity at a point in terms of sequences, open sets and closed sets and examples. Algebra of continuous real valued functions on a metric space. Continuity of composite function. Continuous image of compact set is compact, Uniform continuity in a metric space, examples (emphasis on $\mathbb{R}$). Results such as: every continuous functions from a compact metric space is uniformly continuous. Contraction mapping and fixed point theorem. Applications.

Unit II: Connected spaces (15L)

Separated sets- Definition and examples. Connected and disconnected sets. Connected and disconnected metric spaces. Results such as: A subset of $\mathbb{R}$ is connected if and only if it is an interval. A continuous image of a connected set is connected.
Characterization of a connected space, viz. a metric space is connected if and only if every continuous function from X to $\{1, -1\}$ is a constant function. Path connectedness in $\mathbb{R}^n$, definition and examples. A path connected subset of $\mathbb{R}^n$ is connected, convex sets are path connected. Connected components. An example of a connected subset of Rn which is not path connected.

Unit III : Sequence and series of functions(15 lectures)

Sequence of functions - pointwise and uniform convergence of sequences of real- valued functions, examples. Uniform convergence implies pointwise convergence, example to show converse not true, series of functions, convergence of series of functions, Weierstrass M-test (statement only). Examples. Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous function, conditions under which integral and the derivative of sequence of functions converge to the integral and derivative of uniform limit on a closed and bounded interval (statements only). Examples. Consequences of these properties for series of functions, term by term differentiation and integration(statements only). Power series in $\mathbb{R}$ centered at origin and at some point in $\mathbb{R}$, radius of convergence, region (interval) of convergence, uniform convergence, term by-term differentiation and integration of power series, Examples. Uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions, the basic properties of these functions.

Reference books:

1. R. R. Goldberg; Methods of Real Analysis; Oxford and International Book House (IBH) Publishers, New Delhi.
2. S. Kumaresan; Topology of Metric spaces.
3. E. T. Copson; Metric Spaces; Universal Book Stall, New Delhi, 1996.
4. Robert Bartle and Donald R. Sherbert; Introduction to Real Analysis; Second Edition, John Wiley and Sons.

Other references:

1. W. Rudin; Principles of Mathematical Analysis.
2. T. Apostol; Mathematical Analysis; Second edition, Narosa, New Delhi, 1974
3. E. T. Copson; Metric Spaces; Universal Book Stall, New Delhi, 1996.
4. P. K. Jain. K. Ahmed, Metric Spaces. Narosa, New Delhi, 1996.
5. W. Rudin, Principles of Mathematical Analysis; Third Ed, McGraw-Hill, Auckland, 1976.
6. D. Somasundaram, B. Choudhary; A first Course in Mathematical Analysis. Narosa, New Delhi
7. G. F. Simmons; Introduction to Topology and Modern Analysis, McGraw-Hi, New York, 1963.
8. Sutherland. Topology.

Course: Intergral Transforms(Elective A)
Course Code: USMT6A4/ UAMT6A4

Unit I: The Laplace Transform (15L)

Definition of Laplace Transform, theorem, Laplace transforms of some elementary functions, Properties of Laplace transform, LT of derivatives and integrals, Initial and final value theorem, Inverse Laplace Transform, Properties of Inverse Laplace Transform, Convolution Theorem, Inverse LT by partial fraction method, Laplace transform of special functions: Heaviside unit step function, Dirac-delta function and Periodic function.

Unit II: The Fourier Transform

Fourier integral representation, Fourier integral theorem, Fourier Sine & Cosine integral representation, Fourier Sine & Cosine transform pairs, Fourier transform of elementary functions, Properties of Fourier Transform, Convolution Theorem, Parseval's Identity.

Unit III: Applications of Integral Transforms

Relation between the Fourier and Laplace Transform. Application of Laplace transform to evaluation of integrals and solutions of higher order linear ODE. Applications of LT to solution of one dimensional heat equation & wave equation. Application of Fourier transforms to the solution of initial and boundary value problems, Heat conduction in solids (one dimensional problems in infinite & semi infinite domain).

Reference Books:

1. Lokenath Debnath and Dambaru Bhatta, Integral Transforms and their Applications, CRC Press Taylor & Francis.
2. I. N. Sneddon, Use of Integral Transforms, Tata-McGraw Hill.

3. L. Andrews and B. Shivamogg, Integral Transforms for Engineers, Prentice Hall of India.

Course: Number Theory and its applications II (Elective B)
Course Code: USMT6B4/ UAMT6B4

Unit I: Quadratic Reciprocity (15 L)

Quadratic residues and Legendre Symbol, Gauss's Lemma, Theorem on Legendre Symbol $\left(\dfrac{2}{p}\right)$, the result: If p is an odd prime and a is an odd integer with $(a, p) = 1$ then $\left(\dfrac{a}{p}\right) = (-1)^t$ where $t = \sum_{k=1}^{\frac{p-1}{2}} \left[\dfrac{ka}{p}\right]$, Quadratic Reciprocity law. Theorem on Legendre Symbol $\left(\dfrac{3}{p}\right)$. The Jacobi Symbol and law of reciprocity for Jacobi Symbol. Quadratic Congruences with Composite moduli.

Unit II: Continued Fractions (15 L)

Finite continued fractions. Infinite continued fractions and representation of an irrational number by an infinite simple continued fraction, Rational approximations to irrational numbers and order of convergence, Best possible approximations. Periodic continued fractions.

Unit III: Pell's equation, Arithmetic function and Special numbers (15 L)

Pell's equation $x^2 - dy^2 = n$, where d is not a square of an integer. Solutions of Pell's equation (The proofs of convergence theorems to be omitted). Arithmetic functions of number theory: $d(n)$ (or $\tau(n)$,) $\sigma(n), \sigma_k(n), \omega(n)$) and their properties, $\mu(n)$ and the Möbius inversion formula. Special numbers: Fermat numbers, Mersenne numbers, Perfect numbers, Amicable numbers, Pseudo primes, Carmichael numbers.

Reference Books:

1. Niven, H. Zuckerman and H. Montogomery; An Introduction to the Theory of Numbers; John Wiley & Sons. Inc.
2. David M. Burton; An Introduction to the Theory of Numbers; Tata McGraw-Hill Edition.
3. G. H. Hardy and E.M. Wright; An Introduction to the Theory of Numbers; Low priced edition; The English Language Book Society and Oxford University Press, 1981.
4. Neville Robins; Beginning Number Theory; Narosa Publications.
5. S. D. Adhikari; An introduction to Commutative Algebra and Number Theory; Narosa Publishing House
6. N. Koblitz; A course in Number theory and Crytopgraphy. Springer.
7. M. Artin; Algebra. Prentice Hall.
8. K. Ireland, M. Rosen; A classical introduction to Modern Number Theory. Second edition, Springer Verlag.

9. William Stalling; Cryptology and network security.

Course: Graph Theory and Combinatorics (Elective C)
Course Code: USMT6C4 /UAMT6C4

Unit I: Colorings of graph (15L)

Vertex coloring- evaluation of vertex chromatic number of some standard graphs, critical graph. Upper and lower bounds of Vertex chromatic Number- Statement of Brooks theorem. Edge colouring- Evaluation of edge chromatic number of standard graphs such as complete graph, complete bipartite graph, cycle. Statement of Vizing Theorem. Chromatic polynomial of graphs- Recurrence Relation and properties of Chromatic polynomials. Vertex and edge cuts, vertex and edge connectivity and the relation between vertex and edge connectivity. Equality of vertex and edge connectivity of cubic graphs. Whitney's theorem on 2-vertex connected graphs.

Unit II: Planar graph (15L)

Definition of planar graph. Euler formula and its consequences. Non planarity of $K_5; K_(3; 3)$. Dual of a graph. Polyhedran in $\mathbb{R}^3$ and existence of exactly five regular polyhedron- (Platonic solids) Colorabilty of planar graphs- 5 color theorem for planar graphs, statement of 4 color theorem. flows in Networks, and cut in a network- value of a flow and the capacity of cut in a network, relation between flow and cut. Maximal flow and minimal cut in a network and Ford-Fulkerson theorem.

Unit III: Combinatorics (15L)

Applications of Inclusion Exclusion Principle- Rook polynomial, Forbidden position problems. Introduction to partial fractions and Newton's binomial theorem for real power series, series expansion of some standard functions. Forming recurrence relation and getting a generating function. Solving a recurrence relation using ordinary generating functions. System of Distinct Representatives and Hall's theorem of SDR.

Recommended Books.

1. Bondy and Murty; Grapgh Theory with Applications.
2. Balkrishnan and Ranganathan; Graph theory and applications.
3. Douglas B. West, Introduction to Graph Theory, 2nd Ed., Pearson, 2000
4. Richard Brualdi; Introduction to Combinatorics.

Additional Reference Book.

1. Behzad and Chartrand; Graph theory.
2. Choudam S. A.; Introductory Graph theory; 3 Cohen, Combinatorics.

Course: Operations Research (Elective D)
Course Code: USMT6D4 / UAMT6D4

Unit I: Linear Programming-I (15L)

Prerequisites: Vector Space, Linear independence and dependence, Basis, Convex sets, Dimension of polyhedron, Faces.
Formation of LPP, Graphical Method. Theory of the Simplex Method- Standard form of LPP, Feasible solution to basic feasible solution, Improving BFS, Optimality Condition, Unbounded solution, Alternative optima, Correspondence between BFS and extreme points. Simplex Method – Simplex Algorithm, Simplex Tableau.

Unit II: Linear programming-II (15L)
Simplex Method – Case of Degeneracy, Big-M Method, Infeasible solution, Alternate solution, Solution of LPP for unrestricted variable. Transportation Problem: Formation of TP, Concepts of solution, feasible solution, Finding Initial Basic Feasible Solution by North West Corner Method, Matrix Minima Method, Vogel's Approximation Method. Optimal Solution by MODI method, Unbalanced and maximization type of TP.

Unit III: Queuing Systems (15L)
Elements of Queuing Model, Role of Exponential Distribution. Pure Birth and Death Models; Generalized Poisson Queuing Model. Specialized Poisson Queues: Steady- state Measures of Performance, Single Server Models, Multiple Server Models, Self- service Model, Machine-servicing Model.

Reference for Unit III:

1. G. Hadley; Linear Programming; Narosa Publishing, (Chapter 3).
2. G. Hadley; Linear Programming; Narosa Publishing, (Chapter 4 and 9).
3. J. K. Sharma; Operations Research; Theory and Applications, (Chapter 4, 9).
4. J. K. Sharma, Operations Research, Theory and Applications.
5. H. A. Taha, Operations Research, Prentice Hall of India.

Additional Reference Books:

1. Hillier and Lieberman, Introduction to Operations Research.
2. Richard Broson, Schaum Series Book in Operations Research, Tata McGrawHill Publishing Company Ltd.

Course: Practicals (Based on USMT601 / UAMT601 and USMT602 / UAMT602)
Course Code: USMTP07 / UAMTP07
Suggested Practicals (Based on USMT601 / UAMT601):

1. Limit continuity and derivatives of functions of complex variables.
2. Steriographic Projection , Analytic function, finding harmonic conjugate.
3. Contour Integral, Cauchy Integral Formula ,Mobius transformations.

4. Taylors Theorem , Exponential , Trigonometric, Hyperbolic functions.
5. Power Series , Radius of Convergence, Laurents Series.
6. Finding isolated singularities- removable, pole and essential, Cauchy Residue theorem.
7. Miscellaneous theory questions.

Suggested Practicals (Based on USMT602 / UAMT602)

1. Examples of rings (commutative and non-commutative), integral domains and fields
2. Units in various rings. Determining characteristics of rings.
3. Prime Ideals and Maximal Ideals, examples on various rings.
4. Euclidean domains and principal ideal domains (examples and non-examples)
5. Examples if irreducible and prime elements.
6. Applications of division algorithm and Eisenstein's criterion.
7. Miscellaneous Theoretical questions on Unit 1, 2 and 3.

Course: Practicals (Based on USMT603 / UAMT603 and USMT6A4 / UAMT6A4 OR USMT6B4 / UAMT6B4 OR USMT6C4 / UAMT6C4 OR USMT6D4 / UAMT6D4)
Course Code: USMTP08 / UAMTP08

Suggested practicals Based on USMT603 / UAMT603:

1 Continuity in a Metric Spaces

2 Uniform Continuity, Contraction maps, Fixed point theorem

3 Connected Sets , Connected Metric Spaces

4 Path Connectedness, Convex sets, Continuity and Connectedness

5 Pointwise and uniform convergence of sequence functions, properties

6 Point wise and uniform convergence of series of functions and properties

7 Miscellaneous Theory Questions.

Suggested Practicals based on USMT6A4 / UAMT6A4

1 Find the Laplace transform of differential and integral equations.

2 Find the inverse Laplace transform by the partial fraction method.

3 Find the Fourier integral representation of given functions.

4 Find the Fourier Sine / Cosine integral representation of given functions.

5 Solve higher order ODE using Laplace transform.

6 Solve one dimensional heat and wave equation using Laplace transform. Solve initial and boundary value problems using Fourier transform.

7 Miscellaneous Theory Questions.

Suggested Practicals based on USMT6B4 / UAMT6B4

1 Legendre Symbol.

2 Jacobi Symbol and Quadratic congruences with composite moduli.

3 Finite continued fractions.

4 Infinite continued fractions.

5 Pell's equations and Arithmetic functions of number theory.

6 Special Numbers.

7 Miscellaneous Theoretical questions.

Suggested Practicals based on USMT6C4 / UAMT6C4

1 Coloring of Graphs

2 Chromatic polynomials and connectivity.

3 Planar graphs

4 Flow theory.

5 Application of Inclusion Exclusion Principle, rook polynomial. Recurrence relation.

6 Generating function and SDR.

7 Miscellaneous theoretical questions.

Suggested Practicals based on USMT6D4 / UAMT6D4
All practicals to be done manually as well as using software TORA / EXCEL solver.

1 LPP formation, graphical method and simple problems on theory of simplex method

2 LPP Simplex Method

3 Big-M method, special cases of solutions.

4 Transportation Problem

5 Queuing Theory; single server models

6 Queuing Theory; multiple server models

7 Miscellaneous Theory Questions.

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

(8) Scheme of Evaluation

Scheme of Examination (75:25)

The performance of the learners shall be evaluated into two parts.

- Internal Assessment of 25 percent marks for each paper.
- Semester End Examination of 75 percent marks for each paper.

I. **Internal Evaluation of 25 Marks:**

T.Y.B.Sc. :

(i) One class Test on unit I of 20 marks of duration one hour to be conducted during Practical session.
Paper pattern of the Test:
Q1: Definitions/ Fill in the blanks/ True or False with Justification (04 Marks).
Q2: Multiple choice 5 questions. (10 Marks: 5×2)
Q3: Attempt any 2 from 3 descriptive questions. (06 marks: 2×3)

(ii) Active participation in routine class: 05 Marks.

OR

Students who are willing to explore topics related to syllabus, dealing with applications historical development or some interesting theorems and their applications can be encouraged to submit a project for 25 marks under the guidance of teachers.

T.Y.B.A. :

(i) One class Test on unit I of 20 marks to be conducted during Tutorial session.
Paper pattern of the Test:
Q1: Definitions/ Fill in the blanks/ True or False with Justification (04 Marks).
Q2: Multiple choice 5 questions. (10 Marks: 5×2)
Q3: Attempt any 2 from 3 descriptive questions. (06 marks: 2×3)

(ii) Journal : 05 Marks.

OR

Students who are willing to explore topics related to syllabus, dealing with applications historical development or some interesting theorems and their applications can be encouraged to submit a project for 25 marks under the guidance of teachers.

II. **Semester End Theory Examinations :** There will be a Semester-end external Theory examination of 75 marks for each of the courses USMT501/UAMT501, USMT502/UAMT502, USMT503 and USMT5A4 OR USMT5B4 OR USMT5C4 OR USMT 5D4 of Semester V and USMT601/UAMT601, USMT602/UAMT602, USMT603 and USMT6A4 OR USMT6B4 OR USMT 6C4 OR USMT 6D4 of semester VI to be conducted by the University.

1. Duration: The examinations shall be of $2\frac{1}{2}$ Hours duration.
2. Theory Question Paper Pattern:

a) There shall be FOUR questions. The first three questions Q1, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The fourth question Q4 shall be of 15 marks based on the entire syllabus.
b) All the questions shall be compulsory. The questions Q1, Q2, Q3, Q4 shall have internal choices within the questions. Including the choices, the marks for each question shall be 30-32.
c) The questions Q1, Q2, Q3, Q4 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.

III. **Semester End Practical Examinations :**
There shall be a Semester-end practical examinations of three hours duration and 100 marks for each of the courses USMTP05/UAMTP05, USMTP06/UAMTP056 of Semester V and USMTP07/UAMTP07, USMTP08/UAMTP08 of semester VI.

In semester V, the Practical examinations for USMTP05/UAPTP05 and USMTP06/UAMTP06 are conducted by the college.

In semester VI, the Practical examinations for USMTP07/UAMTP07 and USMTP08/UAMTP08 are conducted by the University.
Question Paper pattern:

Paper pattern: The question paper shall have two parts A, B.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. ($8 \times 3 = 24$ Marks)

Section II Problems: Attempt any Two out of Three. ($8 \times 2 = 16$ Marks)

Practical Course	Part A	Part B	Marks out of	duration
USMTP05/UAMTP05	Questions from USMT501/UAMT501	Questions from USMT502/UAMT502	80	3 hours
USMTP06/UAMTP06	Questions from USMT503/UAMT503	Questions from USMT504/UAMT504	80	3 hours
USMTP07/UAMTP07	Questions from USMT601/UAMT601	Questions from USMT602/UAMT602	80	3 hours
USMTP08/UAMTP08	Questions from USMT603/UAMT603	Questions from USMT604/UAMT604	80	3 hours

Marks for Journals and Viva:
For each course USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503, USMT504/UAMT504, USMT601/UAMT601, USMT602/UAMT602 USMT603/UAMT603, and USMT604/UAMT604:

1. Journals: 5 marks.
2. Viva: 5 marks.

Each Practical of every course of Semester V and VI shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the certified journal.

xxxxx